Chapter 1

Q) Determine the total number of handshakes if there are 8 people and each handshake involves exactly 2 of them. The expression 8C2 is equivalent to the total number of handshakes.
Q) If 11 people are to be divided into committees of 2, 3, and 6, how many divisions are possible? (11!)/(2! 3! 6!) = 4620.
Q) Imagine a piece of paper with 4 dots placed on it, such that each dot occupies a unique position. Assume that the dots are labelled - 1, 2, 3, and 4 - that any pair of dots can share a maximum of one connection, and a dot cannot be connected to itself. How many configurations of connections can we make if we had n dots? $2\text{\^{}}(\frac{n(n-1)}{2})$ different configurations.

Chapter 2

Q) A 3-person sports team consists of 3 unique positions. If a person is chosen at random from each of 3 different such teams, what is the probability of selecting a complete team? 3!/(3^3) What is the probability that all players selected play the same position? 3/(3^3).
Q) A forest contains 17 elk, of which 8 are tagged. Later, 4 of the 17 elk are captured. What is the probability that 3 of the 4 elks are tagged. Theres's 9 not tagged elks. P(A)= ((8C3)(9C1)/(17C4).
Q) A closet contains 11 pairs of shoes. If 7 shoes are randomly selected, what is the probability that there will be (a) no complete pair? (b) exactly 1 complete pair? The closet contains a total of 22 shoes and we can assume that each shoe is equally likely to be picked. In total there are (22C7) ways to pick 7 shoes out of the 22. In order to avoid a pair, lets count the number of ways there are to do that. First, let's count the number of ways you can pick 7 pairs from 11 total. This is equal to (11C7). For each pair, we want to avoid selecting both shoes. This means we are either going to select the left shoe or the right shoe. In other words, there are 27 ways to do that (i.e., for each pair, there are exactly two choices - left and right). To get the probability, we can use the counting definition of probability. *P*(no complete pair)=(11C7) $\cdot\, 2^7$ / (22C7). b)We know there are exactly (22C7) ways to pick 7 shoes, that's the total number. Now how many ways are there to pick one pair from 11 pairs? Well this number is exactly 11. After selecting a pair, we have 5 shoes to select and we must ensure none of these 5 form a pair. So similar to (a), there are (10C5) ways to pick pairs (notice now we have 10 pairs left to choose from), and for each pair there are 25 ways to ensure we either select the left shoe or the right shoe. And so all together we have 11 · 7!/2! · (10C5) · 2^5 / (22C7).
Q) How many vectors x1,...,xk are there for which each xi is a positive integer such that 1 < or equal xi < or equal n and x1<x2<...<xk. Choosing k distinct values from n (nCk).

Chapter 3

Q) Supppose that an ordinary deck of 52 cards is shuffled and the cards are then turned over one at a time until the first odd number (3, 5, 7, 9) appears. Given that the first odd number is the 11th card to appear, what is the conditional probability that the card following it is the a) five of diamonds? b) queen of clubs?
a) Begin by finding P(D|A), the probability that the card following the first odd number is another odd number, given that the first odd number is the 11th card to appear. After the first 11 card have been turned over, the total number of cards remaining in the deck is 41. The total number of odd numbers in the deck is 16 (4x4). Given that the first odd number is the 11th card to appear, after the first 11 cards have been turned over, the number of odd numbers remaining in the deck is 15. P(D|A) = 15/41. P(B|AD), the probability that the card following the first odd number is the five of diamonds, given that the first odd number is the 11th card to appear and the card following the first odd number is another odd number. No information is known about which odd number was the 11th card to appear. So P(B|AD) = 1/16. Now we can find P(B|A) = P(D|A) P(B|AD) = (15/41)(1/16)
b) P(J) = P(J|K)P(K) + P(J|K^C)P(K^C). We can identify that P(J|K), the probability that the card following the first odd number is the queen of clubs given that the queen of clubs appeared among the first 11 cards. P(J|K) = 0. Now consider P(K^C), the probability that the queen of clubs did not appear amond the first 11 cards. The queen of clubs is not an odd number. The total number of cards that are not odd numbers is 36 (52-16). After the first 11 cards, the number of cards remaining in the deck that are not odd numbers is 26 (52-16-10). The probability that the queen of clubs did not appear among the first 11 cards turned over. P(K^C) = 26/36. The probability that the card following the first odd number is the queen of clubs given K^C is P(J|K^C)=1/(52-11)=1/41. The conditional probability that the card following the first off number is the queen of clubs given that the first odd number is the 11th card to appear P(J)=(1/41)(26/36).
Q) Joe is 68% certain that his missing key is in one of the two pockets, 34% in the left, 34% in the right. When the key is in a certain pocket there's a 30% that the search will not find the key. P(L)=P(R)=0.34. P(!S_R|R)=P(U_L|L)=0.3. P(S_RU_L)=P(S_RU_L|R)P(R) + (S_RU_L|!R)P(!R) = P(S_RU_L|R)P(R) + 0 = P(S_R|R)P(R) = (1-0.3)(0.34) = 0.238. P(U_L) = P(U_L|L)P(L)+P(U_L|!L)P(!L) = 0.30(0.34)+1(1-0.34) = 0.3(0.34) + 0.66 = 0.762. Now P(S_R|U_L)=P(S_RU_L)/P(U_L) = 0.238/0.762. b) Find P(S_R|U_L). P(R|U_L)=P(U_L|R)P(R)/P(U_L) = 1(0.34)/0.762 = 0.4462. P(S_R|RU_L) = 0.7 and P(S_R|RU_L)P(R|U_L) = 0.7(0.4462) = 0.3123. We know that P(S_R|!RU_L)=0 so P(S_R|U_L) =P(S_R|RU_L)P(R|U_L) + P(S_R|(!R)U_L)P((!R)U_L) = 0.3123.
Q) Let *Qn* denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that the following equations are true. Find *Q*8 . The formula given is Q_n = 1/2 Q_{n-1} + 1/4 Q_{n-2} + 1/8 Q_{n-3} such that $Q_0 = Q_1 = Q_2 = 1$. The variable Q_n denotes the probability that you do not get 3 consecutive heads in a row in *n* coin tosses.
So obviously $Q_0 = Q_1 = Q_2 = 1$ since it's impossible to get three heads in a row if you don't even toss the coin three times. *Q*4 = • │ • │ • │ • where each • is a slot that can be *H* or *T* . The event Q_4 can happen in multiple ways. *Q*4 = • │ • │ • │ *T* ← (ends with T) = • │ • │ • this has to be *Q*3 (i.e., no 3 heads in a row). *P*(*Q*4) = *Q*3 · *P* (*T*) = 1/2 *Q*3. However, there are OTHER ways that *Q*4 can happen *Q*4 = • │ • │ *H* │ *T* ← (ends with H T) = • │ • th1is now has to be *Q*2. *P*(*Q*4) = *Q*2 · *P*(*H*) · *P*(*T*) = 1/2 · 1/2 *Q*2 = 1/4 *Q*2 and lastly *Q*4 = • │ *H* │ *H* │ *T* ← (ends with H H T) = • this now has to be *Q*1 *H* │ *H* │ *T P* (*Q*4) = *Q*1 · *P*(*H*) · *P*(*H*) · *P*(*T*) = 1/2 · 1/2 · 1/2 *Q*1 = 1/8 *Q*1 And so putting it all together we have *Q*4 = (• │ • │ • │ *T*) + (• │ • │ *H* │ *T*) + (• │ *H* │ *H* │ *T*) and taking the probability we have *P*(*Q*4) = 1/2 *Q*3 + 1/4 *Q*2 + 1/8 *Q*1. Generalizing this, we have *Qn* = 1/2 *Qn*−1 + 1/4 *Qn*−2 + 1/8 *Qn*−3.
Q) Delegates from 10 countries, including Russia, France, England, and the United States, are to be seated in a row. How many different seating arrangements are possible if the French and English delegates are to be seated next to each other and the Russian and U.S. delegates are not to be next to each other? 2x9!-2x2x8!

Chapter 4

Q) The Draft pick question. Let X Choosing the worst team as my ith pick out of 5 teams. P(choosing worst on first) = P(X=1) = 5/15. P(choosing worst on second) = P(X=2) = P(t1 & t5 + t2 & t5+ t3 & t5…)=(1/15)(5/14) + (2/15)(5/13)... P(choosing worst on third X=3) = (1/15)(2/14)(5/12) + (1/15)(3/14)(5/11)+ ... +(2/15)(1/14)(5/12) + (2/15)(3/12)(5/9)+...
Q) An urn contains 15 red, 12 black and 10 green ball. One color is chosen at random and then 6 balls are chosen. Let X be the number of ball that have the chosen colour. a) P(X = 0) = ⅓ (22C6 + 25C6 + 27C9) / 37C6 b) Let X_i equal 1 if the 1th ball selected is of the chosen colour, and let it equal 0 otherwise. Find P{X_1=1}, i = 1, 2, …, 6. P{X_i= 1} = 1/3. Find E[X]: E[X_i] = 1/3 so E[X] = 6 x 1/3 = 2.

Chapter 5

Q) how to know if f could be a density function? If f(x) ≥ 0 for all x & sum of f(x)=1. To check if f(x) ≥ 0, we need draw the table of variations f'(x)=0 → sign f'(lowest) == sign f'(highest)
Q) If 60% of the population of a large community is in facor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain at least 48 people who are in favor. n = 100 & p = 0.6 so E[X] = np = 60 and Var(X) = np(1-p) = 24. P(at least 48) = 1 - P(at most 47) = 1-P(X<47.5) convert X to Z → P(Z < -2.55)
Q) 1000 independent rolls of a fair die will be made. Compute an approximation to the probability that the number 6 will appear between 165 and 170 times inclusively. If the number 6 appears exactly 170 times, find the probability that the number 5 will appear less than 165 times: So our n = 1000 but with the conditional probability our new n = 1000-170 = 830 and the new probabilities are from 1 to 5 so getting a 5 is 1/5 since the 6 was already taken by EXACTLY 170 times before. So new n = 830 and new p = 1/5

Chapter 6

Q) Let f(x, y) = 3/2 xy where 0 ≤ x ≤ 2, 0 ≤ y ≤ 2 , 0 ≤ 3x+3y ≤ 2, and let it equal 0 otherwise. Show that f(x,y) is a joint probability density function. $\int_0^2 \int_0^{\frac{2-3x}{3}} \frac{3}{2} xy \, dy \, dx = 1$
Q) To know if two functions are independent we need to show that $f_X(x)f_Y(y) = f(x,y)$

Chapter 7

Q) Suppose an urn contains n+m balls, of which n are special and m are ordinary. These items are removed one at a time, with each new removal being equally likely to be any of the balls that remain in the urn. The random variable Y, equal to the number of balls that need to be withdrawn until a total of r special balls have been removed. The expected number of balls that need to be withdrawn until a total of r special balls have been removed is E(Y)=r+mr/(n+1).
Q) There are 4 different types of coupons, the first 2 of which comprise one group and the second 2 another group. Each new coupon is obtained is type i with p_i , where $p_1=p_2$ = 3/8 and $p_3=p_4$=1/8. Find the expected number of coupons that one must obtains to have at least of a) all 4 types: 437/35 b) all of the first group: 4 c) all of the second group: 12, d) all the types of either group: 123/35. → E[X]= $\sum_i \frac{1}{pi} - \sum_{i<j} \frac{1}{pi+pj} + \sum_{i<j<k} \frac{1}{pi+pj+pk} + \ldots + (-1)^{n+1} \frac{1}{p1+\ldots+pn}$

Q) There are two misshapen coins in a box; their probabilities for landing on heads when they are flipped are, 0.4 and 0.6. One of the coins is to be randomly chosen and flipped 13 times. Given that two of the first three flips landed on heads, what is the conditional expected number of heads in the 13 flips? E[N_{13}|F=2] = E[F+N_{10}|F=2] = 2 + E[N_{10}|F = 2] =2 + E[N_{10}|C=c_1] P(C=c_1|F=2) + E[N_{10}|C=c_2] P(C=c_2|F=2). Calculating P(C=c_1|F=2) = $\frac{P(F=2|C=c1)\,P(C1)}{P(F=2|C=c1)P(C=c1)+P(F=2|C=c2)P(C=c2)} = \frac{(3C2)(0.4)^2(0.6)}{(3C2)(0.4)^2(0.6)+(3C2)(0.6)^2(0.4)} = \frac{0.288}{0.288+0.432} = 0.4\ldots$

Chapter 1

Order → Permutation: n!/((n-r)!) No Order → Combination: n!/(r!(n-r)!)	Balls & Bins Question where n is the number of balls and r is the number of bins $\binom{n+r-1}{r-1}$	$P(E\|F) = \frac{P(EF)}{P(F)}$

$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ $\sum_{x=1}^{n} (2x - 1) = n^2$

Chapter 4

Expected Value & Variance

$E[X] = \sum xp(x)$	$E[aX + b] = aE[X] + b$	$E[g(X)] = \sum g(x_i)p(x_i)$	$Var(X) = E[(X-\mu)^2] = \sum (x-\mu)^2 p(x)$	$Var(X) = E[X^2] - (E[X])^2$	$Var(aX+b) = a^2 Var(X)$

Binomial | Negative Binomial | Poisson

$P(X=k) = (nCk)p^k(1-p)^{n-k}$	$E[X] = np$	$Var(X) = np[(n-1)p+1] - (np)^2 = np(1-p)$	$P(X = n) = (n-1)C(r-1)p^r(1-p)^{n-r}$	$E[X]=r/p$ $Var(X) = \frac{r(1-p)}{p \cdot p}$	$P(X=i) = e^{-\lambda}\frac{\lambda^i}{i!}$ OR $e^{-\lambda t}\frac{\lambda t^i}{i!}$	$E[X] = Var(X) = \lambda$

Geometric Random Variable | Hypergeometric Random Variable

$P(X = n) = (1-p)^{n-1}p$	$E[X] = \frac{1}{p}$ $Var(X) = \frac{1-p}{p^2}$	$P(X=i) = \frac{(mCi)\,[(N-m)C(n-i)]}{NCn}$ $E[X] = \frac{nm}{N}$ $Var(X) = np(1 - \frac{n-1}{N-1})$

Chapter 5

Expected Value | Uniformly Distributed Random Variable | CDF for Uniform

$E[X] = \int_{-\infty}^{\infty} xf(x)dx$	$f(x) = \frac{1}{b-a}$ when $a \le x \le b$ and 0 otherwise	$E[X] = \frac{a+b}{2}$ $Var(X) = \frac{(b-a)^2}{12}$	$F(x) = 0$ when $x \le a$, $\frac{x-a}{b-a}$ when $a < x < b$, 1 when $x \ge b$

Standard Normal Variable | Exponential Random Variable | Other Formulas

$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma)^2}$	$Z = \frac{X-\mu}{\sigma}$	$f(x) = \lambda e^{-\lambda x}$ when $x \ge 0$, 0 otherwise	$P\{X < a\} = P\{X \le a\} = F(a) = \int_{-\infty}^{a} f(x)\,dx$	Normal approximation to the binomial distribution $P(X < i) = P(X < i - 0.5)$ $P(X >= i) = 1 - P(X < i - 0.5)$ $P(i<=X<=j) = P(i-0.5 < X < j+0.5)$

Chapter 6

Marginal density $p_y(y) = \sum p(x_i, y)$	Total probability $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dydx = 1$	$\int_a^b udv = [uv]_a^b - \int_a^b vdu$	$\int e^{ax}dx = \frac{1}{a}e^{ax}$	$p_{X\|Y}(x\|y) = P(X=x\|Y=y) = \frac{p(x,y)}{p_Y(y)}$	$F_X = \sum p_{X\|Y}(a\|y)$ s.t. $a \le x$

Chapter 7

$Var(\sum x_i) = \sum Var(x_i)$	$E[\sum x_i] = \sum E[x_i]$	$E[XY] = \sum\sum xy\, p(x, y)$	$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$	$Cov(X, X) = Var(X)$	If X and Y are independent then $E[XY] = E[X]E[Y]$

$\rho(X, Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$	$Cov(\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} y_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} Cov(x_i, y_j)$	$Var(\sum_{i=1}^{n} x_i) = \sum_{i=1}^{n} Var(x_i) + 2\sum\sum_{i<j} Cov(x_i, y_j)$	$E[X\|Y=y] = \sum_x xp_{X\|Y}(x\|y)$